Reactive Power Management in Micro Grid with Considering Power Generation Uncertainty and State Estimation

Mohammad Reza Forozan Nasab¹, Javad Olamaei¹*

¹Department of Electrical Engineering, South Tehran Branch, Islamic Azad University, Tehran, Iran.

Abstract
Optimal reactive power dispatch problem in power systems has thrown a growing influence on secure and economic operation, nonlinear and multi-modal problems. Used methods in this issue can be divided into two categories: First, the classical methods like linear programming (LP), nonlinear programming (NLP), quadratic programming (QP), interior point methods (IPM), Newton-based methods, and the second, heuristic methods like genetic algorithm (GA), evolutionary programming (EP), and particle swarm optimization (PSO). In this paper, projected quasi-Newton method (PQN) is used as an optimal algorithm. This algorithm is applied on a 6-bus micro grid in medium voltage level. To make the problem more realistic, a wind turbine is put in one of the buses to consider uncertainty in power generation. Also two buses data are not available to add state estimation to the problem. For troubleshooting of power generation uncertainty, time series prediction model is used to predict wind speed. To overcome the problems of unavailability of some bases information, maximum likelihood weighted least squares estimation (MLWLSE) is used. Finally obtained information is used to optimize the reactive power in micro grid.

Keywords: Projected Quasi-Newton method, micro grid, reactive power optimization, time series prediction model, maximum likelihood weighted least squares estimation.

1. INTRODUCTION
The reactive power optimization has played an important role in optimal operation of power systems. It is a sub-problem of the optimal power flow (OPF) calculation, which adjusts all kinds of controllable variables, such as generator voltages, transformer taps, shunt capacitors/inductors, and handles a given set of physical and operating constraints to minimize transmission losses or other concerned objective functions. It is known that the
reactive power optimization is a non-linear and multimodal optimization problem.

Many conventional techniques such as gradient-based search algorithms and various mathematical programming methods have been proposed to deal with OPF problems.

In the last decades, computational intelligence-based techniques such as genetic algorithm (GA), particle swarm optimization (PSO) and differential evolution (DE) [1] were proposed in reactive power optimization. The quasi-Newton method is a Newton-based approach that uses an approximation of hessian reverse matrix instead of real reverse in Newton method. This estimation changes information in different approaches, form the simplest formation which is constant in whole rehearsal process to advanced formation. The estimations are made based on obtained data during reduction process [2]. The Newton-based methods also widely are used in power systems. Peschon et al [3] are formulized general problem of minimizing the operating cost of a power system by proper selection of the active and reactive productions as a nonlinear programming problem. This general problem is particularized to the minimization of transmission line losses by suitable selection of the reactive productions and transformer tap settings. An efficient computational procedure based on the Newton-Raphson method for solving the power-flow equations and on the dual (Lagrangian) variables of the Kuhn and Tucker theorem is discussed.

Mamandoar and Chenoweth [4] presented a mathematical formulation of the optimal reactive power control (optimal VAR control) problem and gave results from the algorithm tests. The model minimizes the real power losses in the system. The problem was solved by standard techniques. Bjelogrlic et al [5] considered an application of Newton's optimal power flow methods to secondary voltage/reactive power control in transmission networks. An efficient computer program based on sparse matrix/vector techniques was developed for this purpose. Grudinin [6] presented a reactive power optimization model that was based on successive quadratic programming (SQP) methods. An efficient approximation algorithm of initial problem by quadratic programming (QP) problem was described. The quadratic programming problem was solved on the basis of the Newton type quadratic programming method. ZHAO Wei-Xing and LIU Ming-Bo [7] offered a new decomposition algorithm for solving reactive-power optimization problem of multi-area power systems based on Generalized Minimal Residual (GMRES) method and approximate Newton directions.

Burchett and Happ [8] offered an exact optimal power flow model for dispatching generation, transformer taps, and generator voltages to minimize operating costs while guaranteeing a steady state secure operating point after a contingency. A full ac power flow model was used, which permits including voltage and reactive power constraints for optimization. The optimization algorithm solves a sequence of linearly constrained sub-problems using a quasi-Newton search direction. Bolognani and zampieri [9] considered a distributed system of N agents, on which they defined a
quadratic optimization problem subject to a linear equality constraint. Their suggested algorithm was applied for solving the optimal distributed reactive power compensation in smart micro grids. Bruno et al [10] proposed a methodology for unbalanced three-phase OPF (TOPF) for distribution management system (DMS) in a smart grid. In the formulation of the TOPF, control variables of the optimization problem are actual active load demand and reactive power outputs of micro generators. The TOPF is based on a quasi-Newton method and makes use of an open-source three-phase unbalanced distribution load flow.

In this work, projected quasi-Newton method is applied for reactive power optimization in a 6-bus micro grid. One of the buses is connected to a wind turbine with its power generation is variable subject to wind speed and needs to be predicted. Two buses have not enough equipment for measurement needed parameters that they are required for state estimation. This micro-grid is connected to macro grid by slack bus. This paper uses macro grid as a large virtual generator. It means, there are no limitations in active and reactive power supply in slack bus. Totally in this plan, for every ten minutes, the needed parameters are measured or estimated and the data output is used as the data input for the projected quasi-Newton algorithm. Output of quasi-Newton algorithm is the value of net reactive power injected to micro grid network by considering acceptable voltage range for buses that minimize transmission line losses.

2. PROBLEM FORMULATION

As we propose to study the micro grid in medium voltage level, \( \frac{R}{X} \) is small enough and we can use Eq. (1) in order to calculate the transmission line loss [11],

\[
P_L = \sum_{i=1}^{N} \sum_{j=1}^{N} \left[ a_{ij} (P_iP_j + Q_iQ_j) + b_{ij}(Q_iP_j - P_iQ_j) \right]
\]

where \( a_{ij} \) and \( b_{ij} \) are:

\[
a_{ij} = \frac{V_{ij}}{V_iV_j} \cos(\delta_i - \delta_j) \quad (2)
\]

\[
b_{ij} = \frac{V_{ij}}{V_iV_j} \sin(\delta_i - \delta_j) \quad (3)
\]

and the used variables in Eqs. (1)-(3) are defined in following,

\( P_L \): Real power loss in transmission line
\( P_i \): Injected active power of bus \( i \) to network
\( Q_i \): Injected reactive power of bus \( i \) to network
\( V_i \): Voltage of bus \( i \)
\( \delta_i \): Angle of bus \( i \)
\( r_{ij} \): Resistance of transmission line between buses \( i \) and \( j \)
\( N \): Number of transmission lines in network

So the objective optimization function is:

\[
\text{Min} \{ P_L \} \quad (4)
\]

with below constraints:

\[
V_{imin} \leq V_i \leq V_{i\ max} \quad (5)
\]

\[
Q_i \ min \leq Q_i \leq Q_i \ max \quad (6)
\]

Assumption is that the load flow constraints are considered. Also the flow power in lines is always less than thermal
limit. Indexes, “max” and “min” show the the most and the least acceptable values for reactive power and voltage. \( r_{ij} \) is independent variable and it is related to transmission line characteristics. \( P_i \) is independent variable and it is obtained from network. \( Q_i, V_i \) and \( \delta_i \) are dependent variables. The result of the optimization problem is an amount of \( Q_i \) which is in allowed limit and keeps \( V_i \) and \( \delta_i \) in allowed limit and minimize \( P_L \).

3. PROJECTED QUASI-NEWTON METHOD

Consider the following optimization problem with nonlinear equality constraints:

\[
\text{Min } f(x), \\
\text{subject to } c(x) = 0
\]  

(7)

where \( f: \mathbb{R}^n \rightarrow \mathbb{R}^1 \cdot C: \mathbb{R}^m \rightarrow \mathbb{R}^m \cdot m \leq n \) Let \( g(x) = \nabla f(x) \in \mathbb{R}^n \), \( C(x) = (c_1(x), \ldots, c_m(x)) \in \mathbb{R}^m \), and

\[
A(x) = \nabla c(x) = [\nabla c_1(x), \ldots, \nabla c_m(x)] \in \mathbb{R}^{n \times m}
\]

(8)

Assuming that \( A(x) \) has full column rank, one can make a QR-decomposition as,

\[
A(x) = [Y(x), Z(x)] [R(x) \begin{bmatrix} \mathbf{I}_m \end{bmatrix} \mathbf{0}] = Y(x)R(x)
\]

(9)

where \( [Y, Z] \) is an orthogonal matrix, \( R(x) \) is a Nonsingular upper triangular matrix of order \( m \), \( Z(x) \in \mathbb{R}^{n \times t} \), and \( t = n - m \). The column vectors of \( Z(x) \) form an orthonormal basis for the null space \( N(A(x)^T) \), i.e.,

\[
A(x)^T Z(x) = 0
\]

(10)

The columns of \( Y(x) \in \mathbb{R}^{n \times m} \) form a normal orthogonal basis of the range space \( R(A(x)) \) of \( A(x) \). Clearly,

\[
Y(x)^T Y(x) = I_m, \quad Z(x)^T Z(x) = I_t
\]

(11)

\[
Y(x)^T Y(x) + Z(x)^T Z(x) = I_n.
\]

(12)

Let, \( L(x, \lambda) = f(x) - \lambda^T c(x) \)

(13)

Now, we consider the Lagrange problem according to Eq. (7), where \( \lambda \) is the solution vector of the least-square problem, \( \min_{\lambda} [A(x)\lambda - g(x)] \). From Eq. (11), we have,

\[
\lambda(x) = (A(x)^T A(x))^{-1} A(x)^T g(x) = R(x)^{-1} Y(x)^T g(x)
\]

(14)

Let,

\[
w(x, \lambda) = \nabla_{xx}^2 L(x, \lambda)
\]

(15)

be the Hessian matrix of the function \( L(x, \lambda) \) with respect to \( x \). A principal distinction between this method and the usual quasi-Newton methods is that, in the former approach, the updating matrix \( \mathbf{B} \in \mathbb{R}^{t \times t} \) is an approximation of the square matrix \( Z(x)^T \) \( Z(x) \) of order \( t \), whereas in the latter the updating matrices approximates \( w(x, \lambda) \).

For simplicity, we denote \( f(x_k) \) by \( f_k \), \( \nabla f(x_k) \) by \( \nabla f_k \) and \( \nabla^2 f(x_k) \) by \( \nabla^2 f_k \).

In each iteration, this method solves the following equations

\[
R_k^\dagger \rho_k^\dagger = -C_k \quad \text{and}
\]

(16)

\[
B_k \rho_k^z = -Z_k^T g_k
\]

(17)

In order to obtain \( \rho_k^\dagger \) and \( \rho_k^z \), let

\[
\rho_k = Z_k \rho_k^z + Y_k \rho_k^\dagger \quad \text{and}
\]

(18)

\[
x_{k+1} = x_k + \rho_k
\]

(19)

The computation terminates when \( C_k = 0 \) and \( Z_k^T g_k = 0 \). At Such a point, the Karush-
Kuhn-Tucker condition is satisfied. The matrix $B_k$ is updated using the BFGS or DFP formulas after each iteration [12].

4. IMPLEMENTATION OF PQN FOR REACTIVE POWER OPTIMIZATION

Fig. 1 shows a six bus meshed micro grid. The bus number 1 is point of common coupling to the micro grid. This bus lacks generator or load and when connecting to the micro grid, its voltage is one per unit and there are no limitations for active and reactive power supplying through this bus. The bus number 2 is connected to a wind turbine that has uncertainty of power generation. The buses number 3-6 have active and reactive power generators. The active and reactive generators do not influence the subject and only their power limits is constraint to the subject. The buses number 5 and 6 lacks appropriate equipment for accurate measurement of required quantities for network management that needs to state estimation. All of busses except the bus number 1 include without uncertain loads. Also generator sources in micro grid are able to supply all the micro grid loads in islanding mode.

This micro grid has a management center that obtains result of all required quantities by means of the measurement equipment installation in each bus except buses number 5 and 6 that lacks appropriate measurement equipment. This center is able to control the power generation of all buses.

![Fig. 1. Six bus meshed micro grid.](image-url)
in their allowed limits but it has no control on loads. In other words, the loads act independently and the management center only can supply loads by varying on generating power sources. The objective of the management center is to reduce the transmission line loss by controlling reactive power generation of each source and here there is no control on active power. It is assumed that after collecting required information through measurement equipment and estimation of required unmeasured or uncertain quantities, the micro grid management center applies this information to obtain the amount of reactive power generation of buses for minimizing the transmission line loss by means of projected quasi-Newton method. Fig. 2 shows the structure of our proposed algorithm. In order to obtain the amount of reactive power generation of buses, the following steps are being performed.

First, we divide the time into 10 minutes with equal intervals. Regardless of the micro grid being separated from the macro grid or being connected to the macro grid. The network feeds all the micro grid loads. The measurement equipment is transferring information from each bus to the micro grid management center. with the information obtained in $t = [-10,0)$ and before that we use wind speed to compute wind turbine power in $t = [0,+10)$ according to the time series prediction model (TSPM) [14]. Among comparison between different methods, in [14] it was shown that the TSP model alongside with the support vector machine (SVM) regression in 10 minutes intervals had minimum error.
Fig. 3 Structure of modified proposed algorithm according to 3 possible states.
Then, we use the available information before \( t = 0 \) and TSP model output to estimate the required management center parameters on buses which lacks appropriate measurement equipment. Maximum likelihood weighted least squares estimation (MLWLSE)\[15\] in \( t = [0,+10) \) is used.

The injection reactive power of each bus which should be supplied by sources is computed by using projected quasi-Newton and the computed information in \( t = [0,+10) \) by means of MLWLSE as input. Then, sources had to generate the computed amount through management center. These computed values are only valid in \( t = [0,+10) \). Actually, by varying the amount of load consumption and wind speed in time, the system optimum points are displaced and it needs to be updated. For this purpose, we consider \( t = +10 \) as the beginning time, the obtained information from wind speed in \( t = [0,+10) \) and before, is used in TSP model. Also, we use available information before \( t = +10 \) as well as TSP model output, for estimating required management center parameters about buses lacking appropriate measurement equipment, according to the MLWLSE method in \( t = [+10,+20) \). Again by using projected quasi-Newton method and the information computed from state estimation algorithm in \( t = [+10,+20) \) as input, the injection reactive power of each bus which should be supplied by sources, is computed and is applied on the buses sources. This trend is being repeated for every 10 minutes, and the computed points are getting close to the optimum points. This process is applicable in both islanding and non-islanding mode. The duration of input information processing and getting required input for applying on micro grid for every 10 minutes is not considered regarding the small dimensions of micro grids in comparison with macro grid.

Case of malfunction in the proposed algorithm, there could be generally 3 states. The first malfunctioning is that the information buses do not reach the management center because of measurement equipment failure, communication platform interruption and not installing appropriate equipment for measuring. The second malfunctioning happens when the order of management center is not run by buses due to failure in needed equipment to apply center orders in buses or interruption in needed communication platform. Finally, the third malfunctioning state is a combination of the first and the second states, in this case, neither information reaches center from buses nor the center orders, executes in buses. To modify the proposed algorithm in each of states, we do as follows (see Fig. 3).

At first, as mentioned, it is tried to gain appropriate estimations for quantities with no correct data in order to be able to use them in the projected quasi-Newton algorithm. This condition is mentioned to emphasize that if number of buses with no measured data changes, then there will be no damages occurred in the proposed algorithm. In the second condition if we notice that the given orders from center have not been responded in one or more buses then the latest calculated and applied quantities on the bus or buses should be immediately imported as constant numbers in to the quasi-Newton algorithm and new calculated values are applied on the buses.
that are capable to respond to orders to minimize possible loss in lines with new conditions. Actually in this condition derivatives of Lagrangian function get zero than constant reactive power of these buses and they are eliminated from calculation process of algorithm optimization points and only reactive power of controllable remains in the algorithm and the optimum result is calculated for applying. In the third condition after noticing of no response to the order, those calculated values are imported in state estimation of the bus or buses as constant numbers in the quasi-Newton algorithm to calculate optimum values under new conditions. All the mentioned points are true whether in situation connected to macro grid or islanding mode.

5. SIMULATION RESULTS.

In order to evaluate the proposed algorithm, we use it for management of reactive power of micro grid system We consider the micro grid is connected to network with mentioned conditions in previous Section.

In Table 1, transmission lines data are given in the studied micro grid [13], in Table 2, voltage and reactive power constraints are given for each bus and in Table 3, data of each bus is given after prediction of wind turbine power and estimation of unmeasured parameters in t = 0. In Table 4, voltage and also injected net

<table>
<thead>
<tr>
<th>Line No.</th>
<th>R</th>
<th>X</th>
<th>B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0342</td>
<td>0.18</td>
<td>0.0106</td>
</tr>
<tr>
<td>2</td>
<td>0.114</td>
<td>0.60</td>
<td>0.0352</td>
</tr>
<tr>
<td>3</td>
<td>0.0912</td>
<td>0.48</td>
<td>0.0282</td>
</tr>
<tr>
<td>4</td>
<td>0.0228</td>
<td>0.12</td>
<td>0.0071</td>
</tr>
<tr>
<td>5</td>
<td>0.228</td>
<td>0.12</td>
<td>0.0071</td>
</tr>
<tr>
<td>6</td>
<td>0.342</td>
<td>0.18</td>
<td>0.0106</td>
</tr>
<tr>
<td>7</td>
<td>0.114</td>
<td>0.60</td>
<td>0.0352</td>
</tr>
<tr>
<td>8</td>
<td>0.0228</td>
<td>0.12</td>
<td>0.0071</td>
</tr>
<tr>
<td>9</td>
<td>0.0228</td>
<td>0.12</td>
<td>0.0071</td>
</tr>
</tbody>
</table>

Table 1. Line data (values are in P.U) [13].

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>V_{min}</th>
<th>V_{max}</th>
<th>Q_{min}</th>
<th>Q_{max}</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.00</td>
<td>1.00</td>
<td>0.00</td>
<td>infinite</td>
</tr>
<tr>
<td>2</td>
<td>0.95</td>
<td>1.05</td>
<td>-0.065</td>
<td>0.10</td>
</tr>
<tr>
<td>3</td>
<td>0.95</td>
<td>1.05</td>
<td>-0.279</td>
<td>0.05</td>
</tr>
<tr>
<td>4</td>
<td>0.95</td>
<td>1.05</td>
<td>-0.1312</td>
<td>0.15</td>
</tr>
<tr>
<td>5</td>
<td>0.95</td>
<td>1.05</td>
<td>-0.065</td>
<td>0.20</td>
</tr>
<tr>
<td>6</td>
<td>0.95</td>
<td>1.05</td>
<td>-0.065</td>
<td>0.10</td>
</tr>
</tbody>
</table>

Table 2. Constraints (values are in P.U).

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>Real Power Generation</th>
<th>Real Load Demand</th>
<th>Reactive Load Demand</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.00</td>
<td>0.00</td>
<td>0.00</td>
</tr>
<tr>
<td>2</td>
<td>0.09</td>
<td>0.20</td>
<td>0.065</td>
</tr>
<tr>
<td>3</td>
<td>0.28</td>
<td>0.85</td>
<td>0.279</td>
</tr>
<tr>
<td>4</td>
<td>0.59</td>
<td>0.40</td>
<td>0.1312</td>
</tr>
<tr>
<td>5</td>
<td>0.37</td>
<td>0.20</td>
<td>0.065</td>
</tr>
<tr>
<td>6</td>
<td>0.16</td>
<td>0.20</td>
<td>0.065</td>
</tr>
</tbody>
</table>
reactive power to network of each bus is written after optimization. Injected net reactive power is the difference between generative and consumption reactive power in each bus. In Table 5, loss of transmission lines is written before and after optimization where the loss reduction in optimum conditions is considerable. Reached data is for applying to micro grid in $t = [0,+10)$. Also this process is practicable for $t = [+10,+20)$ , and $t = [+20,+30)$.

6. CONCLUSION

In this paper, we proposed a method to manage reactive power in micro grid system considering uncertainty in power generation and incorrect monitoring of some data. The algorithm optimization is based on the projected quasi-Newton method. For considering the uncertainty in power generation, one of the micro grid buses is connected to a wind turbine which predict the power generation. Also, SVM regression for building the 10 minutes time series wind speed prediction model is used. Furthermore, for estimating the unmeasured needed quantities, MLWLSE algorithm is used. Possible failure modes of the proposed algorithm considered and solutions offered. Whole of the mentioned conditions are practicable when micro grid is connected to macro grid or islanding mode. Finally, as example, performance of proposed algorithm for a micro grid while connecting to macro grid is studied and its efficiency is proved for reduction of line loss by controlling and management of reactive power sources in micro grid. According to the convergence rate of projected quasi-Newton method, it is recommended for large scale problems and fast decision making in short time intervals.

**Table 4. Optimal results for reactive power and voltage (values are in P.U).**

<table>
<thead>
<tr>
<th>Bus No.</th>
<th>Voltage</th>
<th>Reactive Power</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.0000</td>
<td>-1.4481</td>
</tr>
<tr>
<td>2</td>
<td>1.0327</td>
<td>-0.0166</td>
</tr>
<tr>
<td>3</td>
<td>1.0270</td>
<td>0.0500</td>
</tr>
<tr>
<td>4</td>
<td>1.0427</td>
<td>0.0229</td>
</tr>
<tr>
<td>5</td>
<td>1.0433</td>
<td>0.0488</td>
</tr>
<tr>
<td>6</td>
<td>1.0470</td>
<td>-0.0014</td>
</tr>
</tbody>
</table>

**Table 5. Line loss before and after optimization (values are in P.U).**

<table>
<thead>
<tr>
<th>Before Optimization</th>
<th>After Optimization</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0522</td>
<td>0.0419</td>
</tr>
</tbody>
</table>

REFERENCES


